

# Hawking radiation via Landauer transport model

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Recently, Nation et al confirmed that fluxes of Hawking radiation energy and entropy from a black hole can be regarded as a one-dimensional (1D) Landauer transport process. Their work can be extended to background spacetimes with gauge potential. The result shows that the energy flux, which is indicated to be equal to the energy-momentum tensor flux, contains not only the contribution of thermal flux but also that of particle flux. We find that the charge can also be regarded as transporting via a 1D quantum channel and the charge flux is equal to the gauge flux.

Keywords: Hawking radiation; entropy; Landauer transport model; gauge potential

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## I. INTRODUCTION

In 1974, Hawking [1] published his striking argument in which black holes are not black but can radiate in the curved spacetimes background by the calculation of Bogoliubov transformation coefficients connecting an initial vacuum far outside the collapsing matter to a final vacuum after the black hole formed. It provides not only a firm foundation to black hole thermodynamics but also a platform to research the quantum gravity. Until now, there are many methods to derive Hawking radiation except that of Hawking, such as path integral [2, 3], trace anomaly [4] or gravitational anomaly [5–14], periodic Green function [15, 16], quantum tunneling [17–27], holographic principle [28, 29], and so on. All of them confirmed a fact that Hawking radiation is the intrinsic property of the event horizon.

According to the initial description of Hawking, the radiation arises from the production of virtual particle pairs spontaneously near the horizon due to the vacuum fluctuation. When the negative energy virtual particle tunnels inwards, the positive energy virtual particle materializes as a real particle and escapes to infinity. However, how does the positive energy particle run? Recently, Nation et al [30] proposed that the positive energy particle escaped to infinity via a 1D quantum channel by making use of the Landauer transport model. They found that the flux of energy was equal to Hawking radiation flux perfectly for both fermions and bosons, indicating that the energy flow is particle statistics independent.

The Landauer transport model was first proposed to study electrical transport in mesoscopic circuits [31, 32], and subsequently used to describe thermal transport [33–36]. In 2000, the phononic quantized thermal conductance counterpart was measured for the first time [37]. For the 1D quantum channel of thermal conductance, it is supposed that there are two thermal reservoirs, which are coupled adiabatically through an effective 1D connection, characterized by temperatures  $T_L$ ,  $T_R$  and chemical potentials  $\mu_L$ ,  $\mu_R$ , where subscripts  $L$  and  $R$  denotes the left and right thermal reservoirs respectively. Because of the temperature difference, the thermal transportation will happen.

The key idea in the work of Nation et al is that the black hole and its environment were viewed as two thermal reservoirs. As temperature in the right thermal reservoir, namely the environment of a black hole, approaches to zero and the scattering effect can be ignored, the energy flow can be obtained using the Landauer transport model, which was shown to be equal to the energy-momentum tensor expectation value of an infinite observer. They also found the energy current and the upper limit of entropy current were same for both bosons and fermions.

In this paper, we intend to extend their work to background spacetimes with chemical potential. We will take the higher dimensional Reissner-Nordström-de Sitter black hole, Kerr-Newman black hole, and 5D black ring as examples to check whether this model is still valid for different geometry. For spacetimes with chemical potential, when the near horizon conformal symmetry is considered, the expectation value of charge flow should be investigated except the flow of energy-momentum tensor. Thus, when the Landauer transport model is researched, one should also explore whether the energy flow contains the contribution of particle flow besides the flux of thermal radiation. It is valuable to explore the relation between the charge flow and gauge flux too.

The remainder of this paper is arranged as follows. In Sec.II, we review Hawking radiation flux and gauge flux from a 2D conformal metric by calculating vacuum expectation values of energy-momentum tensor. In Sec.III, flows of energy and charge of both fermions and bosons are studied from the viewpoint of 1D quantum transport. In Sec.IV, Hawking radiation flux and charge flux from the higher dimensional Reissner-Nordström-de Sitter black hole, Kerr-Newman black hole, and 5D

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black ring are investigated. Finally, our discussion and conclusion are given in Sec.V. Throughout this paper, we use the units  $G = \hbar = c = k_B = 1$ .

## II. HAWKING RADIATION FLUX

For Hawking radiation flux in the spacetimes with horizon, there are several methods to derive it. In the following, we will review the idea of Unruh [38] to calculate the vacuum expectation values of energy-momentum tensor. For any background spacetimes, with nonvanishing chemical potential  $A_t$  in the gauge field, one may reduce it to the 2D form as

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2, \quad (1)$$

where the event horizon satisfies  $f(r_h) = 0$ . For this metric, one can define the null coordinates  $u = t + r^*$ ,  $v = t - r^*$ , by introducing the tortoise coordinate transformation defined as  $dr^* = \frac{1}{f(r)}dr$ , and Kruskal coordinates  $U = -\frac{1}{\kappa}e^{-\kappa u}$ ,  $V = \frac{1}{\kappa}e^{\kappa v}$ , which can induce the corresponding conformal form, respectively

$$ds^2 = -f(r)dudv, \quad (2)$$

$$ds^2 = -f(r)e^{-2\kappa r^*}dUdV, \quad (3)$$

where  $\kappa = \frac{1}{2} \frac{\partial f(r)}{\partial r} |_{r_h}$  is the surface gravity.

It is well-known that the classical Einstein field equation can be derived from the classical action by the minimal variational principle. In the semiclassical theory, this principle is still valid, namely

$$\frac{2}{\sqrt{-g}} \frac{\delta \Gamma}{\delta g^{\mu\nu}} = \langle T_{\mu\nu} \rangle, \quad (4)$$

in which  $\Gamma$  is the effective action with central charge  $c = 1$ . Obviously, one should first give the action in order to get the expectation values of energy-momentum tensor. In the gravitational field with  $U(1)$  gauge field background, the energy-momentum tensor are solved as [39–44]

$$\begin{aligned} \langle T_{\mu\nu} \rangle = & \frac{1}{48\pi} [g_{\mu\nu}(2R - \frac{1}{2}\nabla^\rho S \nabla_\rho S) - 2\nabla_\mu \nabla_\nu S \\ & + \nabla_\mu S \nabla_\nu S] + \frac{e^2}{\pi} (\nabla_\mu B \nabla_\nu B - \frac{1}{2}g_{\mu\nu} \nabla^\rho B \nabla_\rho B), \end{aligned} \quad (5)$$

$$\langle J^\mu \rangle = \frac{e^2}{\pi} \frac{1}{\sqrt{-g}} \epsilon^{\mu\nu} \partial_\nu B, \quad (6)$$

in which  $R$  is the 2D scalar curvature,  $\Delta_g$  is the Laplacian,  $\epsilon^{\mu\nu}$  represents the 2D Levi-Civita tensor, and

$$S(x) = \int d^2y \frac{1}{\Delta_g}(x, y) \sqrt{-g} R(y), \quad (7)$$

$$B(x) = \int d^2y \frac{1}{\Delta_g}(x, y) \epsilon^{\mu\nu} \partial_\mu A_\nu(y). \quad (8)$$

The above equations are introduced by Iso et al [39] recently to get the Hawking radiation flux from the viewpoint of anomaly. In order to get the last values of Eqs.(5) and (6), the vacuum states should be considered. Usually, there are three kinds of vacuum states. Boulware vacuum corresponds to our familiar empty state for large radius outside of the black hole, the renormalized physical quantities are divergent. Hartle-Hawking vacuum corresponds to a black hole in unstable equilibrium with infinite sea of black body radiation. Therefore both of Boulware vacuum and Hartle-Hawking vacuum can't describe the ingoing and outgoing modes felicitously. However this problem can be overcome by the Unruh vacuum, in which the ingoing vacuum is defined with respect to the advanced null coordinate  $\frac{\partial}{\partial v}$  and the outgoing vacuum is defined with respect to the Kruskal coordinate  $\frac{\partial}{\partial U}$ . That is to say, in the advanced null coordinate, there is a finite mount of flux at the horizon while no flux at infinity. Meanwhile, there is no flux at the horizon while a finite mount of flux at infinity in the Kruskal coordinate. As the

relations  $J_U = -\frac{J}{\kappa U}$  and  $T_{UU} = (\frac{1}{\kappa U})^2 T_{uu}$  between the retarded null coordinate and Kruskal coordinate are considered, the gauge and Hawking radiation fluxes can be, respectively, written as [39]

$$\langle J_u \rangle = \frac{e^2}{2\pi} [A_t(r) - A_t(r_h)], \quad (9)$$

$$\langle J_v \rangle = \frac{e^2}{2\pi} A_t(r), \quad (10)$$

$$\langle T_{uu} \rangle = \frac{1}{192\pi} (-f'^2 + 2ff'') + \frac{e^2}{4\pi} [A_t(r) - A_t(r_h)]^2 + \frac{f^2(r_h)}{192\pi}, \quad (11)$$

$$\langle T_{vv} \rangle = \frac{1}{192\pi} (-f'^2 + 2ff'') + \frac{e^2}{4\pi} A_t^2(r). \quad (12)$$

From Eq.(9) and Eq.(10), we find the infinite observers will observe a mount of gauge flux  $-\frac{e^2}{2\pi} A_t(r_h)$  while the free falling observers will see a negative flow  $\frac{e^2}{2\pi} A_t(r_h)$  at the event horizon. Similarly, Eq.(11) indicates that the infinite observers will see a bunch of Hawking radiation flow  $\frac{\pi T_h^2}{12} + \frac{e^2}{4\pi} A_t^2(r_h)$ , in which  $T_h = \frac{\kappa}{2\pi}$  is Hawking temperature, and Eq.(12) indicates the free falling observers will see that a bunch of Hawking temperature flow  $-\frac{\pi T_h^2}{12} + \frac{e^2}{4\pi} A_t^2(r_h)$  drops into the event horizon.

### III. LANDAUER TRANSPORT MODEL

In 1957, Rolf Landauer presented a very intuitive interpretation of electron conduction from the viewpoint of 1D quantum transport [45, 46]. This theory was then extended to the case of quantum thermal transport, where two infinite reservoirs with temperature  $T_L, T_R$  and chemical potential  $u_L, u_R$  respectively are adiabatically connected to each other through a 1D channel. Universally, there are several different distribution functions to describe the thermal equilibrium distribution of particles in reservoir, here we adopt the Haldane's statistics [47]

$$f_g(E) = \{\omega[\frac{(E-u)}{T}] + g\}^{-1}, \quad (13)$$

where the function  $\omega(x)$  satisfies the relation

$$\omega(x)^g [1 + \omega(x)]^{1-g} = e^x, \quad (14)$$

in which  $g = 0, g = 1$  correspond bosons and fermions respectively. In fact, fermions and bosons are the special cases of this statistics theory, it has shown the value of  $g$  also can be taken as  $1/4, 1/3, 1/2, 2, 3, 4$  [48].

For the sake of simplicity in the present investigation, we restrict ourselves to ballistic transport, which means that the channel currents do not interfere with each other [30]. The left (right) components of the single channel energy current hence can be written as

$$\dot{E}_{L(R)} = \frac{T_{L(R)}^2}{2\pi} \int_{x_{L(R)}^0}^{\infty} dx (x + \frac{u_{L(R)}}{T_{L(R)}}) f_g(x), \quad (15)$$

in which  $x_{L(R)}^0 = -\frac{u_{L(R)}}{T_{L(R)}}$ . As done in Ref.[30], we define the zero of energy with respect to the longitudinal component of the kinetic energy. The total energy current is then just  $\dot{E} = \dot{E}_L - \dot{E}_R$ .

We first consider the case of fermions, where the contributions of antiparticles should be considered. According to the viewpoint of Landauer, the maximum energy flow of fermions can be treated as the combination of fermionic particle and antiparticle single channel currents. In this case, as  $x = \frac{(E-u)}{T}, y = \frac{(E+u)}{T}$  are defined, the flow of energy can be expressed as

$$\dot{E}_{L(R)} = \frac{T_{L(R)}^2}{2\pi} \left[ \int_{-\frac{u_{L(R)}}{T_{L(R)}}}^{\infty} dx (x + \frac{u_{L(R)}}{T_{L(R)}}) \frac{1}{e^x + 1} + \int_{\frac{u_{L(R)}}{T_{L(R)}}}^{\infty} dy (y + \frac{u_{L(R)}}{T_{L(R)}}) \frac{1}{e^y + 1} \right]. \quad (16)$$

Due to the existence of chemical potential, the lower limit of the integral isn't zero, so one can not finish the integral directly. To get the final value, we first vary the lower limit of integral, that is

$$\begin{aligned} \dot{E}_{L(R)} = & \frac{T_{L(R)}^2}{2\pi} \left[ \int_0^\infty dx \left( x + \frac{u_{L(R)}}{T_{L(R)}} \right) \frac{1}{e^x + 1} + \int_0^{\frac{u_{L(R)}}{T_{L(R)}}} dx \left( -x + \frac{u_{L(R)}}{T_{L(R)}} \right) \frac{1}{e^{-x} + 1} + \right. \\ & \left. \int_0^\infty dy \left( y + \frac{u_{L(R)}}{T_{L(R)}} \right) \frac{1}{e^y + 1} - \int_0^{\frac{u_{L(R)}}{T_{L(R)}}} dy \left( y + \frac{u_{L(R)}}{T_{L(R)}} \right) \frac{1}{e^y + 1} \right]. \end{aligned} \quad (17)$$

As  $\frac{1}{e^{-x}+1} = 1 - \frac{1}{e^x+1}$  is replaced, Eq.(17) takes the form as

$$\begin{aligned} \dot{E}_{L(R)} = & \frac{T_{L(R)}^2}{2\pi} \left[ \int_0^\infty \frac{x}{e^x + 1} dx + \int_0^\infty \frac{y}{e^y + 1} dy + 2 \frac{u_{L(R)}}{T_{L(R)}} \left( \int_0^\infty \frac{1}{e^x + 1} dx \right. \right. \\ & \left. \left. - \int_0^{\frac{u_{L(R)}}{T_{L(R)}}} \frac{1}{e^x + 1} dx \right) + \frac{u_{L(R)}^2}{2T_{L(R)}^2} \right]. \end{aligned} \quad (18)$$

According to the technique of Landau [49], the convergence rate of  $\frac{1}{e^x+1}$  is very fast, the upper limit of integral  $\frac{u_{L(R)}}{T_{L(R)}}$  therefore can be changed into infinity because  $\frac{u_{L(R)}}{T_{L(R)}} > 1$ . The energy current now can be expressed as

$$\dot{E}_{L(R)} = \frac{T_{L(R)}^2}{2\pi} \left[ \int_0^\infty \frac{x}{e^x + 1} dx + \int_0^\infty \frac{y}{e^y + 1} dy + \frac{u_{L(R)}^2}{2T_{L(R)}^2} \right]. \quad (19)$$

Finishing the integration, we have

$$\dot{E} = \dot{E}_L - \dot{E}_R = \frac{\pi}{12} (T_L^2 - T_R^2) + \frac{1}{4\pi} (u_L^2 - u_R^2). \quad (20)$$

Eq.(20) tells us that the energy current flowing through the 1D system can be divided into two independent components: the flux of particles, which is only related to the particles, and the flux of thermal radiation, which is determined by the temperature of the reservoirs entirely. Our result, obviously, is different from that of Nation et al because of the emergence of chemical potential.

According to the Landauer transport theory, the charge can also be transported by the 1D quantum channel. As the scattering effect is ignored, the charge flux can be expressed as

$$i = \frac{T_{L(R)} e}{2\pi} \int_{-\frac{u_{L(R)}}{T_{L(R)}}}^\infty \frac{1}{e^x + 1} dx. \quad (21)$$

For the case of fermions, the contribution of antiparticle should also be considered as

$$i = \frac{T_{L(R)} e}{2\pi} \int_{-\frac{u_h}{T_{L(R)}}}^\infty \frac{1}{e^x + 1} dx + \frac{T_{L(R)} e}{2\pi} \int_{\frac{u_{L(R)}}{T_{L(R)}}}^\infty \frac{1}{e^y + 1} dy. \quad (22)$$

As the same case in Eq.(19), when the lower limit of integral  $\frac{u_{L(R)}}{T_{L(R)}}$  is changed as infinity, the second term in the right of Eq.(22) will vanish and the first term will lead to

$$i = \frac{e}{2\pi} (u_L - u_R), \quad (23)$$

which shows that the charge current doesn't depend on the temperature and it only relates to the charge and chemical potential.

Now, we check whether the Landauer transport model is valid for the bosons. Putting  $g = 0$  into Eq.(15), the energy current can be expressed as

$$\dot{E}_{L(R)} = \frac{T_{L(R)}^2}{2\pi} \int_{-\frac{u_{L(R)}}{T_{L(R)}}}^\infty dx \left( x + \frac{u_{L(R)}}{T_{L(R)}} \right) \frac{1}{e^x - 1}. \quad (24)$$

After varying the lower limit of integral and adopting  $\frac{1}{e^{-x}-1} = -1 - \frac{1}{e^x-1}$ , the above equation can be rewritten as

$$\dot{E}_{L(R)} = \frac{T_{L(R)}^2}{2\pi} \left[ \int_0^\infty \left( x + \frac{u_{L(R)}}{T_{L(R)}} \right) \frac{1}{e^x - 1} dx - \int_0^{\frac{u_{L(R)}}{T_{L(R)}}} \left( \frac{u_{L(R)}}{T_{L(R)}} - x \right) \frac{1}{e^x - 1} dx - \frac{u_{L(R)}^2}{2T_{L(R)}^2} \right]. \quad (25)$$

Adopting the similar techniques as the case of fermions, we get

$$\dot{E} = \dot{E}_L - \dot{E}_R = \frac{\pi}{12}(T_L^2 - T_R^2) + \frac{1}{4\pi}(u_L^2 - u_R^2), \quad (26)$$

which is consistent with that of fermions.

In the case of system with chemical potential, we have got energy fluxes for both fermions and bosons, which contains the contributions of particles and thermal radiation. We find that they are more complicated, but are still independent on the kinds of particles.

#### IV. HAWKING RADIATION FROM BLACK HOLES VIA LANDAUER TRANSPORT MODEL

In this section, we discuss Hawking radiation of black holes via the Landauer transport model. For this model, one necessity is a heat source. Recall that any spacetimes with horizon will emit Hawking radiation with temperature  $\frac{\kappa}{2\pi}$ , where  $\kappa$  is surface gravity, so the black hole can be treated as one heat source with Hawking temperature on the horizon. When the black hole and its surrounding, which have temperature  $T_L = T_h$ ,  $T_R = 0$  and chemical potential  $u_L = u_h$ ,  $u_R = 0$  respectively, are regarded as thermal reservoirs connected by a 1D quantum tunnel, we find the energy flux and charge flux can be expressed as

$$\dot{E} = \dot{E}_L - \dot{E}_R = \frac{\pi T_h^2}{12} + \frac{u_h^2}{4\pi}, \quad (27)$$

$$\dot{I} = \frac{eu_h}{2\pi}, \quad (28)$$

where we have defined the chemical potential  $u_h = -eA_t(h)$ . Based on above formulisms, we take different background spacetimes as examples to check whether the energy flux and charge flux are equal to the Hawking radiation flux and gauge flux next.

##### A. Higher dimensional Reissner-Nordström-de Sitter black hole

The metric of the higher dimensional Reissner-Nordström-de Sitter black hole with a positive cosmological constant  $\Lambda = n(n+1)/2l^2$  takes the form as [50]

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_n^2, \quad (29)$$

in which

$$f(r) = 1 - \frac{\omega_n M}{r_{n-1}} + \frac{\omega_n Q^2}{2(n-1)V_n r^{2n-2}} - \frac{r^2}{l^2}, \quad \omega_n = \frac{16\pi}{nV_n}, \quad (30)$$

where  $M$  and  $Q$  are the mass and the electric charge of the black hole,  $l$  and  $V_n$  denotes the curvature radius of the de Sitter space and the volume of  $d\Omega_n^2$ , which represents a  $n$ -dimensional spherical hyper-surface of unit radius.

For this spacetime, we can obtain the horizons from  $f(r) = 0$ . Usually, there are three positive roots, which satisfy  $r_- < r_h < r_c$ , and a negative root for  $n \geq 2$ , where  $r_-$ ,  $r_h$ ,  $r_c$  stands for the inner horizon, event horizon, and cosmological horizon respectively. In principle, Hawking radiation can be emitted from both event horizon and cosmological horizon. However, the radiation from cosmological horizon is very weak and the calculation there is similar to the case of event horizon, so we only consider radiation from the event horizon, namely only the event horizon is treated as the thermal source here.

The electric-magnetic gauge potential of the black hole is

$$A_t = -\frac{Q}{(n-1)V_n r^{n-1}}. \quad (31)$$

On the base of the definition of surface gravity, we can also get the Hawking temperature at the event horizon

$$T_h = \frac{\kappa}{2\pi} = \frac{1}{2\pi} \left[ \frac{(n-1)\omega_n M}{2r_h^n} - \frac{\omega_n Q^2}{2V_n r_h^{(2n-1)}} - \frac{r}{l^2} \right]. \quad (32)$$

For the Higher dimensional Reissner-Nordström-de Sitter black hole, we also can reduce it to the 2D metric by dimensional reduction techniques as done in Ref.[39] effectively. Then by the similar skills as in Sec.II, we can get the fluxes of Hawking radiation and charge, which agree with the results in Eqs.(9-12).

From the Landauer transport model, when the event horizon of Higher dimensional Reissner-Nordström-de Sitter black hole is regarded as the heat source, we can get the energy flux and charge flux

$$\dot{E} = \frac{\pi T_h^2}{12} + \frac{e^2 A_t^2(r_h)}{4\pi}, \quad (33)$$

$$\dot{I} = \frac{e^2 Q}{2\pi(n-1)V_n r_h^{n-1}}, \quad (34)$$

in which the chemical potential  $u_h = -eA_t(h)$  is used. Comparing them with Eq.(11) and Eq.(9), we find the energy flux and charge flux are equal to the Hawking radiation flux and gauge flux completely, namely thermal radiation and charge can be transported by a 1D quantum channel. As  $n = 2$  or  $n = 2, l \rightarrow \infty$ , one can also get Hawking radiation flux and charge flux of 4D Reissner-Nordström-de Sitter black hole or 4D Reissner-Nordström black hole from the viewpoint of 1D quantum transport.

### B. Kerr-Newman black hole

As analyzed in Ref.[39], the rotating and charged black hole owns not only the  $U(1)$  gauge symmetry with electromagnetic field but also the induced  $U(1)$  gauge symmetry associated with isometry along the  $\phi$  direction, so one should consider the contributions of both electric charge  $e$  and azimuthal quantum number  $m$  when we calculate the energy-momentum tensor flux and charge flux. In Boyer-Lindquist coordinates, the 4D Kerr-Newman black hole can be expressed as [51, 52]

$$ds^2 = -\frac{\Delta}{\rho^2} (dt - a \sin^2 \theta d\phi)^2 + \frac{\rho^2}{\Delta} (dr^2 + \Delta d\theta^2) + \frac{\sin^2 \theta}{\rho^2} [adt - (r^2 + a^2) d\phi]^2, \quad (35)$$

with  $\rho^2 = r^2 + a^2 \cos^2 \theta$ ,  $\Delta = r^2 - 2Mr + a^2 + Q^2 = (r - r_+)(r - r_-)$ , where  $M$ ,  $Q$ ,  $a$  are the black hole mass, charge and the angular momentum per unit mass,  $r_+ = r_+$  and  $r_-$  denote the outer and inner horizons, which are

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2 - a^2}.$$

The background gauge field is

$$A = \frac{Qr}{r^2 + a^2 \cos^2 \theta} (dt - \sin^2 \theta d\phi). \quad (36)$$

The Hawking temperature can be calculated as

$$T_h = \frac{r_+ - r_-}{4\pi(r_+^2 + a^2)}. \quad (37)$$

To derive Hawking radiation from a Kerr-Newman black hole via anomalous point of view, Ref.[39] have given the effective 2D metric with nonvanishing metric component

$$g_{tt} = -f(r) = -\frac{\Delta}{r^2 + a^2}, g_{rr} = f(r)^{-1}, \quad (38)$$

and the corresponding  $U(1)$  gauge background

$$A_t = -\frac{Qr}{r^2 + a^2} - \frac{a}{r^2 + a^2}. \quad (39)$$

Based on above equations, we also can get the energy-momentum tensor flux and gauge flux by the similar skills as in Sec.II. Note that here, the fluxes of energy-momentum tensor and charge depends not only the electric charge but also the azimuthal quantum number  $m$  because of the difference of spacetime geometry.

Making use of the Landauer transport model, we can also get the energy flux and charge flux from the Kerr-Newman black hole

$$\dot{E} = \frac{\pi T_h^2}{12} + \frac{1}{4\pi} \left( \frac{eQr_h}{r_h^2 + a^2} + \frac{ma}{r_h^2 + a^2} \right)^2, \quad (40)$$

$$\dot{I}_m = \frac{m}{2\pi} \left( \frac{eQr_h}{r_h^2 + a^2} + \frac{ma}{r_h^2 + a^2} \right), \quad (41)$$

$$\dot{I}_e = \frac{e}{2\pi} \left( \frac{eQr_h}{r_h^2 + a^2} + \frac{ma}{r_h^2 + a^2} \right), \quad (42)$$

where we have defined the chemical potential as

$$u_h = \frac{eQr_h}{r_h^2 + a^2} + \frac{ma}{r_h^2 + a^2}. \quad (43)$$

Comparing the fluxes with those obtained from conformal symmetry, we find they are equal too. Apparently, not only thermal radiation and particles, but also the angular momentum of the Kerr-Newman black hole will be transported by a 1D quantum channel.

### C. 5-dimensional black ring

The 5D neutral black ring, which was first found in [53], is a vacuum solution of five-dimensional general relativity. Black ring obeys similar thermodynamics laws as black holes. However, their horizon topology is not spherical as black holes but with topology  $S^1 \times S^2$ . Therefore, it is also significant to check whether this background spacetime is valid for the Landauer transport model.

The 5D neutral black ring is [53]

$$ds^2 = -\frac{F(y)}{F(x)} \left( dt - C(\nu, \lambda) R \frac{1+y}{F(y)} d\psi \right)^2 + \frac{R^2}{(x-y)^2} F(x) \left[ -\frac{G(y)}{F(y)} d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)} d\varphi^2 \right], \quad (44)$$

in which

$$F(\xi) = 1 + \lambda\xi, \quad G(\xi) = (1 - \xi^2)(1 + \nu\xi), \\ C(\nu, \lambda) = \sqrt{\lambda(\lambda - \nu) \frac{1 + \lambda}{1 - \lambda}}.$$

The parameters  $\lambda$  and  $\nu$  are dimensionless, which take value in the range  $0 < \nu \leq \lambda < 1$ , and  $R$  corresponds roughly to the radius of the ring circle. The mass and angular momentum of the black ring relates to these parameters as  $M = 3\pi R^2 \lambda / [4(1 - \nu)]$ ,  $J = \pi R^3 \sqrt{\lambda(\lambda - \nu)(1 + \lambda)} / [2(1 - \nu)^2]$ . The coordinates  $\phi$  and  $\psi$  are two cycles of the black ring,  $x$  and  $y$  take the range as  $-1 \leq x \leq 1$ ,  $-\infty \leq y \leq -1$ . The horizon is located at  $y = y_h = -1/\nu$ .

After the consideration of dimensional reduction, the 2D metric, with gauge charge  $l$  and  $U(1)$  gauge potential

$$A_t(y) = \frac{F(y)}{CR(1+y)}, \quad (45)$$

can be expressed as

$$ds^2 = -f(y)dt^2 + f(y)^{-1}dy^2, \quad (46)$$

in which

$$f(y) = \frac{\sqrt{-F(y)}}{CR(1+y)} G(y). \quad (47)$$

Based on the definition of surface gravity, the Hawking temperature can be written as

$$T_h = \frac{l^2}{4\pi R} \frac{1 + \nu}{\sqrt{\lambda\nu}} \sqrt{\frac{1 - \lambda}{1 + \lambda}}. \quad (48)$$

The energy flux and charge flux from the 5D neutral black ring are

$$\dot{E} = \frac{\pi T_h^2}{12} + \frac{l^2}{4\pi} \left( \frac{F(y_h)}{CR(1+y_h)} \right)^2, \quad (49)$$

$$\dot{I}_l = -\frac{l^2 A_t(y_h)}{2\pi} = \frac{l^2}{2\pi} \frac{F(y_h)}{CR(1+y_h)}. \quad (50)$$

We find the energy flux and charge flux equal to Hawking radiation flux and gauge flux respectively, which means that thermal radiation and particles from black ring also can be transported by a 1D quantum channel. The thermal radiation and particles can be transported from black ring mainly stems from that it also owns an event horizon, which can be treated as the thermal source.

## V. DISCUSSION AND CONCLUSION

Flows of energy and charge from the higher dimensional Reissner-Nordström-de Sitter black hole, Kerr-Newman black hole, and 5D black ring have been studied via the Landauer transport model. Though these black holes have different topological geometry, but all of them own horizons, which can emit Hawking radiation with Hawking temperature, hence the horizons and its environments can be connected by 1D quantum channel as two thermal reservoirs and the Landauer transport model is valid. We find the energy current of fermions and bosons transported by a 1D quantum system can be divided into two independent components: the flux of charge, which is only related to the particles entirely, and the flux of thermal radiation, which are determined by the temperature of thermal reservoir irrespective to the number of particles. The reason for particle statistics independence of Landauer transport model maybe stems from that the mutual cancellation of the group velocity and density of states enter the current formulae [30, 33–36].

It should be stressed that in this paper, the Landauer transport model provides not the method to calculate Hawking radiation but the probable way how Hawking radiation runs, namely by a 1D quantum tunnel.

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